

Externalities and Production Efficiency

Gunnar S. Eskeland

Environmental improvements should be sought from different polluters (public or private, producer or consumer, rich or poor) at the same cost, regardless of the nature of the polluting activity. Under a plausible structure of monitoring costs, emissions standards play a central role.



Summary findings

Eskeland brings together two of government's primary challenges: environmental protection and taxation to generate revenues.

If negative externalities can be reduced not only by changes in consumption patterns but also by making each activity cleaner (abatement efforts), how shall inducements to various approaches be combined? If negative externalities are caused by agents as different as consumers, producers, and government, how does optimal policy combine inducements to reduce pollution?

Intuitively it seems right to tax emissions neutrally, based on marginal damages — no matter which activity pollutes or whether the polluter is rich or poor, consumer or producer, private or public. Eskeland provides a theoretical basis for such simplicity.

Three assumptions are critical to his analysis:

- Returns to scale do not influence the traditional problem of revenue generation.
- Consumers have equal access to pollution abatement opportunities (but he also relaxes this assumption).
- Planners can differentiate policy instruments (emission taxes or abatement standards) by polluting good, and by whether the polluter is a consumer,

producer, or government, but they cannot differentiate such instruments (or commodity taxes) by personal characteristics or make them nonlinear in individual emissions.

Among Eskeland's findings and conclusions: Abatement efforts and consumption adjustments at all stages are optimally stimulated by a uniform emission tax levied simply where emissions occur.

It simplifies things that optimal abatement is independent of whether the car is used by government, firms, or households — for weddings or for work.

It also simplifies implementation that the stimulus to abatement at one stage (say, the factory) is independent of whether it yields emission reductions from the factory or from others (say, from car owners who buy the factory's products).

Finally, ministers of finance and of the environment should coordinate efforts, but they need not engage in each other's business. The minister of environment need not know which commodities are elastic in demand and thus would bear a low commodity tax. The finance minister need not know which commodities or agents pollute or who pays emission taxes.

This paper — a product of Public Economics, Development Research Group — is part of a larger effort in the group to establish principles for public intervention. Copies of the paper are available free from the World Bank, 1818 H Street NW, Washington, DC 20433. Please contact Hedy Sladovich, room MC2-609, telephone 202-473-7698, fax 202-522-1154, email address hsladovich@worldbank.org. Policy Research Working Papers are also posted on the Web at www.worldbank.org/research/workingpapers. The author may be contacted at gskeland@worldbank.org. April 2000. (43 pages)

The Policy Research Working Paper Series disseminates the findings of work in progress to encourage the exchange of ideas about development issues. An objective of the series is to get the findings out quickly, even if the presentations are less than fully polished. The papers carry the names of the authors and should be cited accordingly. The findings, interpretations, and conclusions expressed in this paper are entirely those of the authors. They do not necessarily represent the view of the World Bank, its Executive Directors, or the countries they represent.

Externalities and Production Efficiency

Gunnar S. Eskeland*

The World Bank

The findings, interpretations, and conclusions expressed in this paper are entirely those of the authors. They do not necessarily represent the views of the World Bank, its Executive Directors, or the countries they represent. The paper should not be cited without the permission of the authors.

Correspondence: Gunnar Eskeland, The World Bank, Development Research Group,
1818 H St. N.W., Washington, DC 20433, USA. Phone/fax: 1 202 473 7938/522 1154.
geskeland@worldbank.org.

I. Introduction and Summary

In this paper, we combine two challenges of government: taxation for revenue generation and environmental protection. First-best intuition would say that emissions be taxed neutrally according to their marginal damages, without reference neither to the type of activity that pollutes nor to whether the polluter is rich or poor, consumer or producer, private or public. However, there has yet been no theoretical basis for assuming such simplicity if government has a revenue need that requires distortionary taxation.

Our study builds in particular on two important contributions. First, Diamond and Mirrlees, 1971, demonstrated conditions under which optimal taxation involves production efficiency. Their findings imply that the input-output vector is at the aggregate production frontier, a solution that can be implemented by confronting all producers, private as well as public, with the same producer prices. Thus, a social planner may want to insert distorting tax wedges between consumers, and between the set of consumers and the set of all producers, but not between producers, whether private or public. With an external effect from producers and government, a production efficiency result follows directly from Diamond and Mirrlees' treatment. The environmental good is an additional output (or input) of productive sectors, and shall consequently be provided at the same marginal rates of transformation for the set of producers and government seen as a whole. However, the literature including externalities in models of optimal taxation has focused on cases with only one type of polluting activity or source, thus putting aside the question of production efficiency in environmental protection.

The second contribution on which the present study builds is Sandmo's seminal study "Optimal taxation in the presence of externalities" (Sandmo, 1975). Apart from providing the analytical framework used subsequently -- and in this study -- Sandmo made two findings we shall highlight here. First, he concluded that "even in a world of distortionary taxation" ...there is scope for taxing externality-generating commodities according to the Pigovian principle." Second, he noted that the optimal commodity tax structure is "characterized by what might be called an additivity property; the marginal social damage of commodity m enters the formula for that commodity only..." (m is the externality-creating good) and "the optimal tax rate on the externality creating commodity is a weighted average of two terms, of which the second is the marginal

social damage of commodity m . The first term...is composed of the efficiency terms familiar from the theory of optimal taxation”.

An important aim of the present study is to understand conditions under which optimal taxation – and Sandmo’s framework - requires production efficiency, including in environmental protection, when such protection can take many avenues. Diamond and Mirrlees considered briefly whether production efficiency would hold if there is an external effect between consumers, but then without including environmental protection in the concept of production efficiency. They concluded “it seems quite likely that efficiency will be desired in realistic settings”. The concept of production efficiency tested here is a broader one, since we include the environmental good in the vector proposed to be at the aggregate frontier¹. For Sandmo, the proposition of production efficiency in environmental protection was not at the table, since there was only one polluting activity.

To examine the question of whether production efficiency can include efficiency in the protection of the environment, a key assumption is to include *pollution abatement* as an additional avenue for pollution reductions: a polluting consumer (or producer) may spend resources – say on a filter – to reduce emissions per unit consumed (or produced). This allows us to test a proposition of production efficiency more broadly defined: Under what conditions will marginal costs of abatement – per unit of emission reductions achieved - be equalized across activities and agents?

In our model, the set of polluters is not only producers (as in Cremer et al. 1998, Cremer and Gahvari, 1999), or only consumers (as in Sandmo, 1975, Diamond and Mirrlees, 1971), but comprise consumers, producers and government. Briefly put, we ask whether the social planner would tax emissions from different activities (or from producers, consumers, government) differently.

Our model (Section II of the paper) is simple - a structure with fixed coefficients of transformation between private goods is expanded with an external effect, a public good. The public good is in the outset provided by nature, but is reduced as a negative external effect (we call it pollution) results from consumption and production activities. The model involves five minor modifications to Sandmo’s 1975 model. First,

government, firms and consumers are all polluters. Second, in addition to substitution towards non-polluting goods and services, the model allows the polluter to expend resources on *pollution abatement* to reduce emissions. This term includes efforts such as the consumer's maintenance of her car, the producer's installation of a catalytic converter in her product or a filter in her smokestack, modifications of practices or of compounds such as fuels and detergents, and finally cleanup efforts. Third, we allow multiple polluting goods, or activities (we use the word activity to comprise consumption *and* production). Fourth, we allow nonuniformity across polluters in how much they pollute per unit of activity (more precisely, they differ in their costs of pollution abatement). Finally, these modifications themselves invite expansions of the set of policy instruments relative to those allowed by Sandmo and subsequent authors. On the one hand, emission taxes no longer are mere extensions of commodity taxes when pollution abatement is possible (this distinction is also used by Cremer and Gahvari, 1999). Also, we show, standards for abatement (or for emission per unit) can play a role under plausible restrictions on the observability of emissions.

Several studies have provided approaches preparing the ground for this treatment. Bovenberg and van der Ploeg, 1994, introduce abatement, but as public production rather than related to own emissions (a good example might be a municipal wastewater treatment plant). Their discussion is centered on how increased environmental concern influences provision of public goods and consumption of private goods. Goulder et al., 1998, allow abatement amongst producers, and focus on the interaction between environmental instruments and pre-existing taxes. Both these studies employ assumptions giving the labor/leisure choice, not only the environmental good, a particular role in preferences. Cremer et al. 1998 analyze optimal taxation and focus on the interaction between the environmental tax and other instruments, much in the same way as did Atkinson and Stiglitz, 1976, for the interaction between direct and indirect instruments in the traditional problem without externalities. Cremer and Gahvari, 1999, closest to the questions asked here, allow several polluting industries with uniform technology. Amongst their findings is a uniform emissions tax, implicitly showing how Diamond and

¹Since they were proposing an external effect from consumers to consumers, it was quite natural in their context not to expand with the environmental good the vector of inputs and outputs proposed to be at the aggregate frontier.

Mirrlees' production efficiency result must apply if the input or output of producers is expanded with one element - the environmental good.

In section III, we characterize optimal policy assuming equal access to technology for all consumers. We investigate when instruments such as emission taxes will be applied 'neutrally' as expected according to Pigovian principles, including under plausible restrictions on the monitoring of emissions². These results can be viewed as generally extending those of Sandmo, 1975. Also, they extend the results of Eskeland, 1994, on the combination of emission standards and presumptive Pigovian taxes (levied on inputs and outputs), to a case in which taxation is costly.

For polluting producers, production efficiency applies as expected even when firms differ in their access to abatement technology (section III). In section IV of the paper, we introduce nonuniform emission functions for consumers. If the planner can differentiate emission taxes across polluting activities, when are the emission taxes equal across activities, and equal to the one applied to producers? We find that the 'one tax' breaks down if the pattern of nonuniformity across consumers in emission functions lends itself to nonenvironmental goals of the planner, such as to redistribute, or to minimize the distortionary effects of taxation. Certain covariance formulas identify these cases. In the case when emission taxes are not available, presumptive Pigovian taxes on goods and emission standards are no longer 'first-best' when emission functions are nonuniform, so results are modified for that reason. While this result is new in a setting of distortionary taxation, it naturally extends results from a literature examining indirect Pigovian instruments when the externality generating good is itself unavailable or imperfect as a base for a corrective instrument³.

II. The Model

We introduce some variations to existing treatments of optimal taxation in the presence of externalities (Sandmo, 1975, in particular, but also Cremer et al, 1998). The importance of these variations lie in their practical relevance, and we will thus intersperse the text with some examples for illustration.

² As is the tradition in the literature, we use the words tax and taxation whether the rate is positive or negative (in everyday use, the negative rates would be called subsidies).

³ Some notable contributions are: Diamond, 1973; Sandmo, 1976; Balcer, 1980; Wijkander, 1985; Greenwald and Stiglitz, 1986.

Preferences

As is the tradition in the public finance literature, we analyze a setting in which consumers have preferences over private goods as well as a public good (with several public goods, results extend straightforwardly). As a matter of terminology, a term such as ‘a public bad’ could be used for pollution, but we shall generally try to describe the social planner as providing (or procuring) a public good when using taxes or regulation to stimulate pollution reductions. It is sometimes convenient to speak of goods in general, and then let a vector of quantities include pollution as a public good, even though consumers prefer less pollution to more.

Let H denote the set of consumers, and let h be a consumer, $h \in H$. h ’s utility depends on her consumption x_j^h of a set N of market goods, $j = 0, 1, \dots, n$, as well as on a pollution indicator, e :

$$(1) \quad u^h = u^h(x_0^h, x_1^h, \dots, x_n^h, e).$$

We assume that the utility function is continuous, twice differentiable, quasiconcave, and that $u_e^h \leq 0$ ⁴. In addition, we shall assume that preferences are separable between the basket of market goods, $j \in N$, and pollution, so that the marginal rates of substitution between market goods are independent of pollution levels. A sufficient condition for this to hold is that individual preferences can be described by a separable utility function: $u^h = u^h(\psi^h(x_0, x_1, \dots, x_n), e)$. In the literature on taxation in the presence of external effects, separability is typically assumed.⁵

In assuming that pollution is experienced at the same level by all consumers, we combine two properties. The important one is that pollution (or its absence) is a pure public good in the sense of Samuelson, 1954, that there is no rivalry in its consumption. If one person enjoys the low level of pollution, this does not reduce another person’s

⁴ Whenever possible without risking confusion, we shall use subscripts to denote partial derivatives.

$u_e^h \leq 0$ is necessary for us to use words such as ‘negative external effects’ and ‘pollution’, but the results are equally applicable also to positive external effects. An equilibrium in which Pigovian taxation makes distortionary taxation unnecessary is less plausible with positive externalities.

⁵ An assumption of separability between leisure and other private goods is often also included (Cremer et al., 1998, Cremer and Gahvari, 1999, Bovenberg and Goulder, 1996, Bovenberg and van der Ploeg, 1994). In the traditional model without externalities, this additional separability assumption renders (other)

enjoyment. The less significant implied property is homogenous dispersion (which James Meade, 1952, termed atmospheric pollution). It simplifies notation, by ensuring that the marginal damages from emissions (or the benefits from emission reductions) are independent of who or where the polluter is. If damages per unit of emissions vary, say by location or by stack height, accommodation of this fact must be made, and results extend (the question should be asked, however, of whether instruments can be differentiated accordingly).

Emissions and pollution abatement

In the world we try to capture with our model, emissions of pollution are caused by several activities, in production stages as well as consumption stages. Examples that we all know of are that emissions are caused in the production of gasoline, cars, and detergents, and also as households and firms use car services and do their laundry. Also, *pollution abatement*, or efforts to make each activity less polluting, may be undertaken by producers or consumers. Reduced emissions from cars, for instance, can result as the manufacturer changes his product by adding a catalytic converter, as the refinery changes the gasoline characteristics, and as the driver drives more carefully, buys a 'cleaner' gasoline, and improves her maintenance. As these examples illustrate, efforts to abate emissions may well be exerted at a production stage even if emissions occur later, for instance in consumption.

The traditional treatment of externalities in the theoretical literature has been to view substitution in consumption (towards non-polluting goods and services) as the only way to reduce pollution (Cremer and Gahvari, 1999, made advances beyond this). In such a case, it is of no importance whether emissions result from production or consumption – since the assumption of equilibrium ensures that production and consumption move in parallel. Sandmo's important (1975) contribution described emissions as caused by consumption, but with results directly applicable for emissions caused by producers.

In a context with *pollution abatement* in contrast, it could be material whether emissions occur in consumption or in production. To illustrate, if in optimum producers and households face different price vectors and abatement options (they do), are marginal

commodity taxes redundant in the presence of a tax on labor. In a model with a polluting commodity, like Sandmo, 1975, it would then suffice with a tax on labor and an emission tax.

abatement costs in optimum different for car manufacturers and users? Similarly, if a car owner can reduce emissions, should policy stimuli depend on whether she is a consumer, an enterprise or government?

Commodities are potentially polluting in both production and consumption, so we may think of a set of $2n$ *polluting activities*. We choose consumer abatement and emissions as our main presentational vehicle, in part because consumer abatement is novel and poses more interesting questions in a welfare economic perspective. To save on notation, we do not introduce emissions from producers before later in this section.

Individual emissions are caused in association with the consumption of polluting goods and services, as represented by *emission factors* $f_j^h = f_j(b_j^h)$:

$$(2) \quad e_j^h = f_j(b_j^h) \cdot x_j^h \text{ for all goods } j = 1, \dots, n, h \in H,$$

and equivalently for government. (2) reflects that the consumer may expend resources on pollution abatement, b_j^h , in order to reduce f_j^h , the emissions per unit consumed of good j . We assume, until we generalize in section IV, that consumers have access to the same abatement technology, so that emission functions $f_j(b_j^h)$ are uniform across consumers. The assumption that emissions display proportionality with quantity (though conditional on the good in question, and abatement) is restrictive, but allows us to place our results in a literature based on constant returns to scale. To simplify, we assume that the numeraire good is not polluting: $f_o = 0$, and we describe all other goods as polluting:

for $j \neq 0$: $f_j > 0$. Abatement b_j is nonnegative and continuous and f_j is assumed to be continuous and differentiable. We assume that abatement reduces emissions at a decreasing rate, so that the marginal cost of emission reductions $-1/f_{jb}$ is positive and increasing⁶.

The pollution level, the argument in each consumer's utility function, is simply emissions aggregated across polluters and polluting goods⁷:

⁶ The assumption that *all* goods $j = 1, \dots, n$ are polluting simplifies notation. It means that a nonpolluting good is approximated as one for which emissions and emission taxes are trivially low at abatement levels that are trivially low

⁷ We skip important detail here, and some deserve mention: i) the pollution level, here a scalar, may be a vector (concentrations of dust and of ground-level ozone). Extension of results to several 'public goods' (or bads: dust, ozone), with one set of Pigovian taxes for each, is straightforward; ii) whether or not the pollution level is a scalar (say, parts per million of ozone), emissions contributing to the pollution level may

$$(3) \quad e = \sum_{j \in N} \left(\sum_{h \in H} e_j^h + e_j^p \right),$$

where e_j^p denotes emissions resulting if the government uses good j (the generalization with producer emissions is straightforward, and will follow).

*The consumer's problem*⁸

Let t_j and τ_{ej} be linear taxes levied respectively on consumption and emissions of good j , $j = 0, \dots, n$. The consumer faces a price $p_j + t_j$ for each good. $p_o = 1$, $t_o = 0$, so the numeraire good is untaxed. Our model is general in its treatment of private goods, so it is not important whether one thinks of the numeraire good as leisure. In this respect, our model differs from a number of recent contributions on Pigovian taxation in which results are based in part on preferences that are separable in leisure versus other private goods (additional results following from making that assumption are rather obvious)⁹.

We model consumer h as maximizing her utility u^h with respect to consumption and abatement, subject to her budget constraint:¹⁰

$$(4) \quad \text{Max}_{x_j^h, b_j} : u^h(x_0^h, x_1^h, \dots, x_n^h, e) \text{ s.t. } \sum_{j \in N} [p_j + t_j + b_j^h + \tau_{ej} f_j(b_j^h)] x_j^h = 0.$$

In h 's maximization, we shall assume that she considers the level of pollution, e (the sum of what is generated by all polluters), and also public sector revenue to be independent of her own actions. This will be either accurate descriptions or close

be a vector (as with the precursors of ozone: tons emitted of nitrogen oxides and of volatile organic compounds); iii) we abstract from how the pollution indicator translates into damages (health effects, species extinction, corrosion, etc.), when we describe willingness to pay simply as a function of the pollution level.

⁸ We use the terms 'consumer' and 'household' as synonyms, and are thus unable to handle distributional issues within the household. Bergstrom (1989) provides an interesting discussion of when this is appropriate, and of the shortcomings (more recently highlighted in the literature focusing on gender and children).

⁹ In a model with constant transformation coefficients, like ours (see production technology, below), the choice of untaxed commodity is immaterial: it is easily checked that the relative prices obtained here (including those inducing abatement) can be replicated with another choice of untaxed good.

¹⁰ Our budget constraint is consistent with the traditional: $\sum_{j=1}^n (p_j + t_j) x_j = I - l$ where I is

endowment and l is leisure. With $x_o \equiv l - I$ (x_o is a negative figure), we have $\sum_{j=0}^n (p_j + t_j) x_j = 0$.

When we introduce consumer abatement and emission taxes, we obtain

$$\sum_{j=0}^n (p_j + t_j + b_j^h + \tau_{ej} f_j) x_j = 0.$$

approximations if the number of individuals, H , is large. These assumptions are rather natural extensions of the assumptions of competitive equilibrium, under which producers and consumers take prices as given. In the present model, they consider two additional variables as independent of their actions: total pollution and public revenue¹¹.

The first-order conditions for h 's individual optimum are her budget constraint and, for all goods $j=1, \dots, n$:

$$(5) \quad \frac{u_j^h}{u_0^h} = p_j + t_j + b_j^h + \tau_{ej} f_j(b_j^h) \equiv q_j^h \quad \text{and}$$

$$(6) \quad -\frac{1}{f_{jb}(b_j^h)} = \tau_{ej}.$$

The first equality in (5) shows how the consumer will set marginal rates of substitution between private goods equal to the relative marginal costs of these goods. In the second equation in (5), we have taken advantage of the fact that these marginal costs are independent of consumption levels, so that the marginal cost is also a unit cost, and introduced the symbol q_j^h to represent this 'all-inclusive consumer price'. In (6), the consumer sets her marginal cost of emission reductions equal to the emission tax rate. These marginal costs would be equal across consumers even if emission functions differed across consumers. However, with homogenous emission functions, abatement b_j^h and emission factors f_j^h will also be the same across consumers, ensuring that the all-inclusive consumer prices are uniform across consumers. We shall use this property to suppress individual superscripts for b_j and q_j until section IV, in which we adopt heterogeneous emission functions.

Finally we may sketch a generalization. If emissions occur at production stages as well, and if producers in sector j face emission taxes and abatement opportunities, then the producer price in (5) will itself be a sum components, to include the producer's abatement and taxes on emissions in production. Then, self interested producers will join the consumer in an effort to minimize the all-inclusive consumer price.

¹¹ See Sandmo, 1975, or Eskeland, 1994, for some further treatment. If there are H individuals who take into account the effect of their own actions, then our approximation error is to set $H/(H-1)$ equal to one.

Production technology

To describe the economy's technological constraint - its ability to transform one bundle of consumption goods into another - let capitalized variables without superscripts denote aggregate quantities: $X_j = \sum_{h \in H} x_j^h + x_j^P$, with x_j^P denoting government

consumption. A rather general description of technology would be:

$$(7) \quad F(X_1, \dots, X_n) = Y_0$$

where Y_0 is work, or endowment less leisure and abatement:

$$(8) \quad Y_0 = I - l - \sum_{j=1}^n b_j X_j = -X_0 - \sum_{j=1}^n b_j X_j.$$

One assumption embodied in (7) and (8) is that the damages from pollution do not affect production possibilities. Thus, the motivation for pollution abatement is found solely in the way pollution affects household utility (1).

Our model of the production side of the economy shall involve additional restrictions: fixed factors of transformation between market goods (see, for instance, Sandmo, 1975, or Cremer et al., 1998):¹²

$$(9) \quad \sum_{j=1}^n c_j X_j = Y_0,$$

where the vector c consists of the constant transformation coefficients. Though we have used aggregate quantities, we may think of (9) as describing a generally available conversion technology, possessed and controlled by many independent producers. When these producers compete in input and output markets, each handling their share of the aggregate quantities, profits will be zero and producer prices will be equal to marginal costs:

$$(10) \quad p_j = c_j, \text{ all } j=1, \dots, n, \text{ and } p_0 = 1.$$

III. Optimal taxation

A benevolent planner

¹² The assumption of fixed coefficients of transformation – or of constant producer prices – is motivated by our desire to compare with standard results in the optimal taxation literature. Diamond and Mirrlees, 1971, showed that the results based on constant producer prices apply also to the more general case of constant returns to scale.

Let a benevolent planner's objectives be represented by a welfare function defined over individual utility levels, $w = w(v^1, v^2, \dots, v^h)$, where $v^h = v^h(q^h, I^h, e)$ is the indirect utility function corresponding to (1), and q^h is given by p, t, τ , as described by (5). He maximizes welfare subject to a constraint that revenues are equal to a predetermined minimum - enough to finance exogenous public sector expenditures¹³ (we initially assume that government abatement is exogenously given). Apart from the instruments used here, we assume that the planner does not have available other instruments for redistribution or revenue mobilization. However, our analysis applies also to the case when there are other instruments with redistributive and revenue implications. Such other instruments could be uniform poll taxes as well as non-linear income taxes¹⁴.

Commodity taxes are in the literature typically restricted to be linear, with a brief justification being that the planner's information includes aggregate quantities $\sum_h x_j^h$, but not the H vector x_j^h . Thus, the tax man may observe liters of gasoline exiting the refinery gate or the gas station, but not individual purchases to an extent attributable to individual consumers. We may add that nonlinear commodity taxes (or commodity taxes differentiated by personal characteristics) would involve not only costly information and administration, but also distortions, as consumers would engage in costly exchange of goods and services. Thus, the restriction that commodity taxes be linear can rest on a broader set of considerations than only information availability.

This latter, broader justification is more appropriate when we assume that emission taxes may be differentiated by polluting commodity (gasoline versus heating oil), but are confined to be linear and uniform across consumers. We may think of emissions as in principle observable by the planner at the emitting source (a meter on each car, for instance, displaying the car's cumulative emissions at year-end), but that the attribution of emissions to households would be costly and lead to distortions under

¹³ Individual budget constraints add up to the technology constraint (9) if we include that of the planner:

$\sum_h \sum_j (t_j + \tau_{ej} f_j) x_j^h = \sum_j ((p_j + b_j^P) x_j^P)$, where the right hand side is public expenditures.

¹⁴ The analysis applies by viewing the presented sufficient conditions as a subset of conditions for optimal policy. In the tradition of Mirrlees (1971), nonlinear income taxes are introduced by assuming that individuals differ in endowment of time in productivity units, but that only income (work times wage) is observed and taxable by the planner. One approach is to assume a discrete number of types and nonlinear income taxes subject to self-selection constraints. Cremer et al., 1998, demonstrate analysis of instruments such as emission taxes in a broader context which includes non-linear taxes.

nonlinear taxation.¹⁵ Apart from these considerations, our assumptions are motivated by our practical aim of checking whether optimal emission taxes would apply neutrally when they can be differentiated by commodity - a question which is less well defined for non-linear instruments.

The Lagrangian of the planner's maximization problem is:

$$(11) \quad L_{t_i, \tau_{ej}} = w(v^1, \dots, v^h) + \mu \sum_{j \in N} \left\{ \sum_{h \in H} (t_j + \tau_{ej} f_j(b_j)) x_j^h - (p_j + b_j^p) x_j^p \right\}, i=1, \dots, n.$$

Optimal taxation

To simplify exposition, we introduce the following definitions:

$$\beta^h = \frac{\partial w}{\partial v^h} \frac{\partial v^h}{\partial t_i} \quad \text{and} \quad \alpha^h \equiv - \frac{\partial v^h}{\partial e} / \frac{\partial v^h}{\partial t_i}.$$

β^h is the marginal value of additional income to individual h as valued in optimum by planner's welfare function. α^h is h 's willingness to pay – in terms of the numeraire good - for pollution reductions, a non-negative number by assumption.¹⁶

We shall initially consider government abatement as given. Partially differentiating (11) with respect to the n commodity tax rates and the n emission tax rates, first order conditions for optimal taxation are, for all $i = 1, \dots, n$:

$$(12) \quad \frac{\partial L}{\partial t_i} = 0 \Leftrightarrow - \sum_{h \in H} \beta^h [x_i^h + \alpha^h \frac{de}{dt_i}] + \mu \sum_{g \in H} \left[x_i^g + \sum_{j \in N} (t_j + \tau_{ej} f_j) \frac{dx_j^g}{dt_i} \right] = 0, \text{ and}$$

¹⁵ Examples to illustrate such difficulties: Multiple car households, (temporary) exchange of cars and car services, multihousehold heating and municipal waste-water discharge. The impossibility of using potential information on individual emissions for nonlinear taxation is particularly clear in the case of polluting producers in a model with constant returns to scale, as ours. Nonlinear taxation of emissions would be ruled out by the costless replication (or merger) of firms. For households, on the other hand, non-linear taxation of emissions could with plausibility be feasible and attractive, save for reasons of administrative difficulty and distortions. We learn something relevant for non-linear taxation of household emissions when we treat differences in emission factors in section IV.

¹⁶ Use the definition of the indirect utility function and the envelope theorem:

$$u(x(q, I, e), e) = v(q, e, I) \Leftrightarrow - \frac{\partial u}{\partial e} = - \frac{\partial v}{\partial e}. \quad \text{Divide by minus the marginal utility of income to express}$$

$$\text{willingness to pay in terms of the numeraire: } - \frac{\partial u}{\partial e} / \sum_j \frac{\partial u}{\partial x_j} \frac{\partial x_j}{\partial t_i} = - \frac{\partial v}{\partial e} / \frac{\partial v}{\partial t_i}.$$

$$\frac{\partial \mathcal{L}}{\partial \tau_{ei}} = 0 \Leftrightarrow$$

$$(13) \quad - \sum_{h \in H} \beta^h (f_i x_i^h + \alpha^h \frac{de}{d\tau_{ei}}) + \mu \sum_{g \in H} [f_i x_i^g + \tau_{ei} f_{ib} b_{ir} x_i^g + \sum_{j \in N} (t_j + \tau_{ej} f_j) \frac{dx_j^g}{d\tau_{ei}}] = 0.$$

In (12), we have used the fact that producer prices are independent of commodity taxes:

$$\frac{dp_j}{dt_i} = 0, \Rightarrow \frac{\partial^h}{\partial \alpha_i} = \sum_j \frac{\partial^h}{\partial q_j} \frac{dq_j}{dt_i} = \frac{\partial^h}{\partial q_i}, \text{ and Roy's identity: } \frac{\partial^h}{\partial q_i} = -x_i^h \frac{\partial^h}{\partial \mathcal{A}^h}. \text{ In (13), we}$$

have used $\frac{dq_i}{d\tau_{ei}} = f_i$, and $\frac{dq_j}{d\tau_{ei}} = 0$, which follow from differentiation of q_i (see

equation 5) and the envelope theorem. To develop these expression further, we may use:

$$(14) \quad \frac{de}{dt_i} = \sum_j \sum_g f_j (b_j) \frac{dx_j^g}{dt_i} \text{ and } \frac{de}{d\tau_{ei}} = \sum_g \left(f_{ib} b_{ir} x_i^g + \sum_j f_j \frac{dx_j^g}{d\tau_{ei}} \right).$$

The $2n$ equations (12) and (13), with (14) describe an optimal tax structure for the n commodity tax rates and the n emission tax rates.

The uncompensated demand functions are in general defined over prices, income, and the quantity of the public good, $x_j^g = x_j^g(q, I^g, e)$. Let us now employ the assumption of separability between pollution and other goods: $x_{je}^g = 0 \Rightarrow$

$$(15) \quad dx_j^g / dt_i = x_{ji}^g \text{ and } dx_j^g / d\tau_{ei} = x_{ji}^g f_i, \text{ all } i, j, g.$$

When using (15) and (14), (12) and (13) simplify to, for all $i = 1, \dots, n$:

$$(16) \quad - \sum_{h \in H} \beta^h \left[x_i^h + \alpha^h \sum_{g \in H} \sum_{j \in N} f_j x_{ji}^g \right] + \mu \sum_{g \in H} \left[x_i^g + \sum_{j \in N} (t_j + \tau_{ej} f_j) x_{ji}^g \right] = 0, \text{ and}$$

$$(17) \quad - \sum_{h \in H} \beta^h \left[f_i x_i^h + \alpha^h \sum_{g \in H} \left(f_{ib} b_{ir} x_i^g + \sum_{j \in N} f_j x_{ji}^g f_i \right) \right] \\ + \mu \sum_{g \in H} \left[f_i x_i^g + \tau_{ei} f_{ib} b_{ir} x_i^g + \sum_{j \in N} (t_j + \tau_{ej} f_j) x_{ji}^g f_i \right] = 0.$$

In order to gain further insights into the tax structure implied by (16) and (17), we shall go via simplifying assumptions.

Assumption 1: No abatement available: emission factors are exogenously given

The case with an exogenously given emission factor was analyzed by Sandmo, 1975. When polluting goods cannot be made less polluting per unit (in our model's terminology, when $f_{ib} = 0$, all $i \in N$ for any b_i), pollution reductions will rely solely on changes in consumption patterns towards goods that are less polluting (say: from motorcycles to bicycles, from cigars to cigarettes). The model with exogenously given emission factors is – fortunately – unrealistic in most practically interesting cases, but provides important insights even for the more general case¹⁷. We show it here as a basis for comparison with existing literature, and also to generalize to several polluting activities (or goods).

With no abatement available, the equations in (6) do not apply, and every equation in (17) is simply f_i times the corresponding equation in (16). Thus, the $2n$ by $2n$ coefficient matrix is at most of rank n and at most n instruments are required to implement the optimal allocation. Assuming that the n equations in (16) are linearly independent, we use this redundancy to set emission taxes all equal to zero and implement the optimal solution using commodity taxes only (as in Sandmo's treatment). Substituting $\tau_{ej} = 0$, all $j \in N$ into (16), we obtain

$$(18) \quad \sum_{j \in N} t_j \sum_{g \in H} x_{ji}^g = - \sum_{h \in H} \left[x_i^h \left(\frac{\beta^h}{\mu} - 1 \right) + \frac{\alpha^h \beta^h}{\mu} \sum_{j \in N} f_j \sum_g x_{ji}^g \right], \text{ all } i \in N.$$

Insights are gained by rearranging to have tax rates on the left hand side. We display the solution for the case with four goods (0,1,2,e), two tax rates:¹⁸

$$(19) \quad t_1 = \frac{\sum_h \left(\frac{\beta^h}{\mu} - 1 \right) (x_1^h \overline{x_{22}} - x_2^h \overline{x_{21}})}{H(x_{11} \overline{x_{22}} - x_{21} \overline{x_{12}})} + f_1 \frac{\sum \beta^h \alpha^h}{\mu} \text{ and}$$

¹⁷ It is not unrealistic in *all* interesting cases, and realism depends on the level of generality in the model. In the example of CO₂, there are virtually no abatement technologies available to users if we examine (fuel) efficient combustion technologies for each fuel (say: coal fired power plant). Thus, for a model disaggregating to the individual fuel, the assumption of fixed emission factors would be quite appropriate. In contrast, for a model with an energy aggregate only, one could represent the flexibility within this aggregate (towards fuels that are less CO₂ intensive) as abatement options.

¹⁸ In the more general case, we have $t_k = \sum_h ((\beta^h / \mu) - 1) \sum_i x_i^h F_{ik} / |E| + f_k \sum_h \beta^h \alpha^h / \mu$, where E is the coefficient matrix in (18), and F_{ik} is the cofactor of row i , column k .

$$t_2 = \frac{\sum_h \left(\frac{\beta^h}{\mu} - 1 \right) (x_2^h \bar{x}_{11} - x_1^h \bar{x}_{12})}{H(\bar{x}_{11} \bar{x}_{22} - \bar{x}_{21} \bar{x}_{12})} + f_2 \frac{\sum \beta^h \alpha^h}{\mu},$$

which is the solution given by Sandmo, 1975. In (19), we have used consumer averages ($\bar{x}_{ij} = \sum_h x_{ij}^h / H$) to highlight which terms in these formulas weighted by β . The optimal tax formula ‘cares’ about individual consumption x^h and willingness to pay α^h , but about demand responsiveness only in aggregate.

Sandmo describes the optimal tax structure (19) as giving commodity taxes in the presence of externalities an ‘additivity property’ (page 92). Of the two terms, the first is equal to the formula for optimal commodity taxes in the traditional problem with no external effects (see below), and the second is motivated by the need to correct for external effects. The term for the corrective tax is, as Sandmo noted, zero for commodities that are not polluting, and it is zero for all commodities if there is no willingness to pay for pollution reductions ($\sum_h \beta^h \alpha^h = 0$). The addition to Sandmo’s result given here is only that with several polluting goods, the corrective tax element in each tax formula is uniform per unit of public good (the emission factor in each formula ensures this). This result, it can be argued, follows so directly from Sandmo’s analysis, it is implicit.

We shall now use the redundancy in tax instruments to explore a specific alternative way to implement this allocation. Let us examine a solution including the following tax rate levied on emissions uniformly across polluting goods:

$$(20) \quad \tau_{ek} = \tau_{el} \equiv \tau_e = \frac{\sum \beta^h \alpha^h}{\mu}, \text{ all } k, l = 1, \dots, n.$$

Substituting (20) into (16), we have:

$$(21) \quad -\sum_h \beta^h x_i^h + \mu \sum_g [x_i^g + \sum_j t_j x_{ji}^g] = 0, \text{ all } i = 1, \dots, n.$$

(20) and (21) also solves (17), so this system of commodity taxes and a uniform emission tax implements the optimal allocation. Solving for the commodity tax rates, and again assuming two taxed goods, (21) is satisfied for¹⁹:

¹⁹ The more general case corresponds to the formula in footnote 19, eliminating the Pigovian element.

$$(22) \quad t_1 = \frac{\sum_h \left(\frac{\beta^h}{\mu} - 1 \right) (x_1^h \overline{x_{22}} - x_2^h \overline{x_{21}})}{H(x_{11}x_{22} - x_{21}x_{12})} \quad \text{and} \quad t_2 = \frac{\sum_h \left(\frac{\beta^h}{\mu} - 1 \right) (x_2^h \overline{x_{11}} - x_1^h \overline{x_{12}})}{H(x_{11}x_{22} - x_{21}x_{12})}.$$

The formulas for the commodity taxes in (21) and (22) (and also those for the non-Pigovian terms in (18), (19)) are equivalent to the expressions for the solution to the traditional problem of optimal commodity taxation without pollution (i.e. Samuelson, 1951), but the actual tax rates in models with and without pollution will in general not be the same. To highlight the implied structure for the non-Pigovian part, let us follow Samuelson and use the Slutsky equation and the symmetry of the compensated demand derivatives $s_{ij}^h(q, u^h) = s_{ji}^h(q, u^h)$ to see that (21) \Rightarrow

$$(23) \quad \sum_j t_j \sum_h s_{ij}^h = \sum_h x_i^h \left[\sum_j t_j \frac{\partial x_j^h}{\partial I^h} + \left(\frac{\beta^h}{\mu} - 1 \right) \right], \quad \text{all } i = 1, \dots, n.$$

As Samuelson pointed out, if one assumes an arbitrarily small revenue requirement and identical consumers, (23) gives the same proportionate reduction in compensated demand for all commodities. Sandmo (1976) highlighted that this feature of (23) extends to hold for substantive revenue requirements if all taxed goods have equal income elasticities. Simplifications often used to illustrate the implications of (21) (or 23) are to assume that the displayed cross price responses are zero, implying that taxes are inversely proportional to own-price elasticities.²⁰ The structure is also equivalent to the one analyzed by Corlett and Hague, 1953, who showed that with two taxed goods the good be taxed at a higher rate which has a higher degree of complementarity with the untaxed good.

Thus, it can be seen, the standard and recognized results for optimal commodity taxes extend to the case with an environmental externality, as long as the externality is

²⁰ Samuelson's (1951) rule should in part be attributed to F. P. Ramsey, 1927, who finds: "the production of each commodity should be diminished in the same proportion". As noted in Munk, 1978, Ramsey's changes in production are equal to changes in uncompensated demand, equal to those of compensated demands if income elasticities are zero. Samuelson writes: "Aspects of the right answer have been hinted at by Ramsey (1927)". Diamond, 1975, proposes to use the concept social marginal utility of income,

γ^h , rather than our β^h , with $\gamma^h = \beta^h + \mu \sum_j t_j \frac{\partial x_j^h}{\partial I^h}$. This is intuitively an equally attractive concept and simplifies expression of certain results: "for each good the change in aggregate compensated demand is proportional to the covariance between individual quantities demanded and social marginal utility of income" (page 338). A good orientation in this literature is provided by Auerbach, 1985.

taken care of by an appropriate tax levied on emissions. This is in itself not an interesting observation, first because there is redundancy in instruments²¹, and second because the formal equivalence of commodity tax formulas with and without presence of external effects in no way would imply equivalence in rates. However, there are two aspects of this solution giving the emission tax an intuitive interpretation as a price. First, uniformity across polluting activities hint that emission reductions are elicited at the same marginal cost wherever they can be found - alluding to a simple procurement rule. Second, the expression itself consists of a weighted sum of the willingness to pay for emission reductions, reminiscent of the Samuelson (1954) rule for optimal provision of public goods.

Definition: When the purpose is to internalize external effects such as emissions, we shall use term *Pigovian tax* in the traditional way – to mean a corrective tax - if the tax/subsidy is levied directly on emissions (or more generally on a measured contribution to the public good). We shall use the term *presumptive Pigovian tax* if the corrective tax is levied on a commodity (such as an input or an output in the externality generating activity) with the rate per unit of the commodity motivated by an emission factor, in presumption of emissions²².

Proposition 1: Fixed emission factors and presumptive Pigovian taxation

With fixed emission factors, two alternative tax structures implementing the optimal allocation are

- a) as in Sandmo, 1975, a commodity tax structure in which the formula is the sum of presumptive Pigovian taxes and the formula for optimal commodity taxes in the traditional problem without external effects (equation 19, or more generally from 18).*
- b) a combination of a Pigovian tax (20) uniformly applied to emissions from all polluting goods and services, and a commodity tax structure satisfying the formula*

²¹ See Cremer et al., 1998, who operate with nonlinear instruments more general than ours.

²² Terms and definitions: We thus associate the term *Pigovian tax* with the objective of internalizing externalities, but not a rule or a level (contrasting, for instance, Cremer et al., 1998). An *indirect Pigovian tax* typically means a corrective tax levied not on the externality causing good itself, but on substitutes and complements (Sandmo, 1976b). Outside the realm of Pigovian taxation, *indirect taxes* have a different meaning (see, for instance Atkinson and Stiglitz, 1976). The term *presumptive* is for income taxes

for optimal commodity taxes in the traditional problem without external effects (equation 22 or more generally from 21).

The proof is given above.

A historical note is worthwhile. Ramsey wrote in the introduction to his 1927 treatment of the traditional problem without externalities: “I shall suppose that, in Professor Pigou’s terminology, private and social net products are always equal, or has been made so by State interference not included in the following.” Sandmo, 1975 followed up to solve the twin tasks thus referred to by Ramsey. In his concluding paragraphs on how to assess real-world taxes, Ramsey wrote: “In the case of motor taxes we must separate off so much of the taxation as is offset by damage to the road. This part should be so far as possible equal to the damage done. The remainder is a genuine tax and should be distributed according to our theory;”. Thus, we may say Ramsey had in mind something like Sandmo’s ‘additivity property’. Another aspect in Sandmo’s formula was that the damage component (reflecting benefits of public good provision) is adjusted by the shadow price of public revenue. This adjustment points back to an important idea of Pigou’s, that when revenue generation has its own costs “expenditure ought not to be carried so far as to make the real yield of the last unit of resources expended by the government equal to the real yield of the last unit left in the hands of the representative citizen.”²³ Pigou’s conjecture later was found to require qualification, but the indicated adjustment is assured if there is separability between the public good and taxed goods (our model) and taxed goods are not predominantly inferior goods (See Atkinson and Stern, 1973).

Assumption 2: Endogenous emission factors

When abatement technologies are available for a set M of polluting goods ($f_{ib} < 0, i \in M$), the system (16) and (17) is at most of rank $n+m$. For simplicity, let us assume that the rank is $2n$ so all n polluting goods have abatement technologies.

established, with a meaning parallel to ours for corrective taxes (See, for instance Musgrave and Musgrave, 1984, or articles in Newbery and Stern, 1984, and Gillis, 1989).

²³ Pigou, 1947, 1949 reprint, page 34.

We may immediately substitute the Pigovian tax (20) into (16) and (17) to see that this again reduces the set of equations to one of rank n , and a formula equal to the one defining the optimal commodity tax structure in the traditional problem without external effects²⁴. Thus, we may conclude with:

Proposition 2: Endogenous emission factors and Pigovian taxation

A combination of a Pigovian tax (20) and a commodity tax structure satisfying the formula for optimal commodity taxes in the traditional problem without external effects (equation 22, or more generally 21) is optimal also in the context of endogenous emission factors.

One way of this looking at this result is that one introduces a good-specified emission tax to induce abatement. First, this tax is to be the same across polluting goods. Second, in a context of commodity taxes satisfying the formula for optimal taxes in the traditional problem without externalities, this emission tax also induces optimal substitution towards cleaner goods and services. Another way to communicate the result is to suggest that the presumptive Pigovian part of Sandmo's formula be replaced - when possible - by a tax levied on emissions. Viewing Sandmo's formula as a sum of two taxes, the presumptive Pigovian tax is transformed to a Pigovian tax by moving the emission factor from the tax rate to the base of a new tax. This scheme is strictly preferred in a context in which abatement can be induced, and thus optimal in a wider range of circumstances.

Assumption 3: Emissions are not observed (or not taxable) at the individual level

We here examine briefly the implications of two crudely defined constraints on the monitoring of emissions. Let us first assume that the planner is not able to tax emissions, but he can regulate abatement and levy commodity taxes. Standards for emission factors (or for abatement) are often seen in the real world, and one interpretation of this is that it is less costly to monitor emission *factors* or technology, than it is continuously to monitor emissions (or to obtain a measure of cumulative emissions, say at year-end)²⁵. Examples

²⁴ In an earlier version of this paper (Eskeland, 1996), direct derivation of this tax structure was provided. It can be made available upon request.

²⁵ The existence of emission standards has been given several interpretations. The interpretation compatible with our treatment here is that monitoring emissions at the source is prohibitively costly, but that abatement

are that vehicles and industries face emission standards, either mandating a particular technology or defining a maximal rate for emissions per unit of output (say: grams per mile or per gallon of fuel, for vehicles)²⁶. Simultaneously, the vehicles may be subject to odometer charges (by mile, or kilometer) or fuel taxes, and industries may be subject to taxes on inputs and outputs.

With these assumptions, the planner has two instruments for each good (t_j , b_j), again a total of $2n$ instruments at his disposal. Modifying the Lagrangian (11) with the applicable budget constraint $\sum_h \sum_j t_j x_j^h = R$ and instruments, the first order conditions for optimum are the budget constraint and, for all $i=1, \dots, n$,

$$(24) \quad - \sum_{h \in H} \beta^h \left[x_i^h + \alpha^h \sum_{g \in H} \sum_j f_j(b_j) x_{ji}^g \right] + \mu \sum_{g \in H} \left[x_i^g + \sum_j t_j x_{ji}^g \right] = 0, \text{ and}$$

$$(25) \quad - \sum_{h \in H} \beta^h \left[x_i^h + \alpha^h \sum_{g \in H} \left(\sum_j f_j(b_j) x_{ji}^g + f_{ib}(b_j) x_i^g \right) \right] + \mu \sum_{g \in H} \left[\sum_j t_j x_{ji}^g \right] = 0.$$

where we have used $\partial q_j / \partial b_j = 1$, $j=1, \dots, n$ (see equation 5).

It is easily checked that if the planner sets abatement or emission standards such that marginal costs equal social benefits adjusted for the shadow price of public revenue:

$$(26) \quad \frac{\sum_{h \in H} \alpha^h \beta^h}{\mu} = \frac{-1}{f_{ib}},$$

(or technology, or emission *factors*) can be monitored cheaply ex ante (as a car model is approved by the authorities) or periodically (at annual vehicle inspections) or even randomly. Such a structure is analyzed in Eskeland, 1994, where it was shown that standards then should be combined with presumptive Pigovian taxes, levied for instance on a car's odometer, or on a variable input, such as gasoline. A more comprehensive practical discussion of these principles in the light of monitoring and enforcement problems is given in Eskeland and Devarajan, 1996. Another important interpretation of standards and regulation in general, as opposed to the tax treatment, is that they give the planner a way to distribute emission permits (See Buchanan and Tullock, 1975, and Baumol and Oates, 1988).

²⁶ Standards for emission factors and for abatement (or technology) have equivalent implications in our model, but more generally instruments should be as open and flexible as possible. Thus, standards will be more effective, *ceteris paribus*, if they define maximum emission factors than if they specify a technology which meets that goal.

For reviews including discussion of monitoring costs and their consequences, see, for example, Eskeland and Jimenez, 1992, and Cropper and Oates, 1996. Using a model with monitoring costs, Schmutzler and Goulder, 1997, conclude "Pure output taxes are optimal under sufficiently high monitoring costs, sufficiently limited options for emission reductions by means other than output reduction, and sufficiently high substitutability of the output".

For cars, a potentially important advantage of odometer charges (relative to fuel taxes) that is not exploited in the literature nor in the real world is that it could implement a system where the corrective tax is raised conditional on vehicle characteristics or emission factors. With fuel taxes, such differentiation will be constrained.

then (25) reduces to (24) times the vector f , to be satisfied if (24) is satisfied. Then, the formula for optimal commodity taxes including presumptive Pigovian taxes (19, or more generally 18) satisfies (24) and (25).

Proposition 3: Emission standards and presumptive Pigovian taxation

- a) If the planner cannot tax emissions, but he can set standards for emission factors (or abatement) and levy commodity taxes, then the optimal allocation is the same as when emission taxes are available. The marginal cost of abatement per unit of emissions is the same across activities, as if driven by optimal emission taxes (20). Commodity taxes will satisfy the formula for optimal commodity taxes including presumptive Pigovian taxes (equation 19, or more generally 18, as in Sandmo, 1975).*
- b) If the planner cannot address abatement in any way, then the optimal allocation is one of commodity taxes including presumptive Pigovian taxes (19 or more generally 18) as in Sandmo, 1975.*

Part *a*) of Lemma 3 (the allocation is the same as with emission taxes) is seen by noticing that abatement is identical, and that such abatement and the level of presumptive Pigovian taxes result in the same all-inclusive consumer prices and the same public revenue. Part *b*) of Lemma 3 is seen by noticing that when the planner has only n commodity tax rates as instruments, optimality is characterized only by the budget constraint and the n equations in (24), equivalent to (13). Under the assumptions of *b*), emission factors are higher and environmental costs (the sum of abatement and Pigovian taxes) weigh more heavily in the ‘all-inclusive consumer price’ than in *a*).

The contribution of Lemma 3 is a modest one, since it is well known that the efficiency properties of a quota for emissions can be the same as those for an emission tax (See, for instance, Baumol and Oates, 1988, or Tietenberg, 1992). What we do here is to introduce a generalizing and a restrictive feature. We generalize by looking at the use of quotas (or standards) in a context with distortionary revenue generation. One of the lessons thus learned is that such a system of optimal standards and presumptive taxes in our model has the same allocative and distributive impacts as a system with emission charges. On the restrictive side, as we generalize to introduce constraints on monitoring and enforcement, we assume that these allow a separate policy instrument to make

activities *cleaner* per unit of activity. ‘Emission quotas’ often come in the form of a standard for emissions per unit of output, a fact that has formerly been afforded scant notice and interpretation in the public finance literature²⁷. A contribution of Eskeland, 1994, was to show that emission quotas of this kind make activities cleaner, but fail to give appropriate incentives to reduced consumption, and thus should be accompanied by a presumptive Pigovian tax²⁸.

Finally, we should emphasize that Sandmo’s 1975 result should be read as holding for any given level of abatement. Building on this, Lemma 3 contributes with optimal abatement.

Assumption 4: Producers and government abate and pollute

Proposition 4: Production efficiency

Optimal abatement is efficient in the sense that the marginal cost of abatement per unit of emissions is the same not only across activities but also across agents: government, households and firms.

Consider first the case of firms, and the simple case in which production of good j involves firms with the same costs, abatement opportunities a_j and consequences $f_j(a_j)$. Let us first assume that abatement in production influences production-stage emissions (i.e. at the car-maker’s smoke-stack, rather than at his customer’s tail-pipe). Then the producer price for good j (see equation 10) will include not only the producer’s costs of abatement but also his emission taxes, $\tau_{aj}f_j$:

$$(27) \quad p_j = c_j + a_j + \tau_{aj}f_j(a_j).$$

²⁷ Important textbooks such as Baumol and Oates, 1988, and Tietenberg, 1992, do not mention monitoring costs as possibly favoring (or explaining) emission standards. *Uncertainty* in estimates of benefits and costs is an accepted consideration in quantity instruments versus prices (Weitzman, 1974), but that argument does not rely on costly monitoring of emissions.

²⁸ One can argue that such emission standards implicitly award emission quotas to operators of polluting processes: You may emit more, but the same amount per unit, if you drive more, or if you produce more steel. That perspective is even more important when existing facilities are ‘grandfathered’ (given more lenient treatment) in regulations. For analysis of such differential treatment, see Crandall et al., 1986, and Harrington, 1997, on automobiles, and Nelson et al., 1993, on EPA’s new source emission standards. Grandfathering has positive and negative connotations: ‘Grandfather clauses allow rents to be shifted to those grandfathered without distorting supply responses’ (Wittman, 1989).

It is easily checked that an emission tax $\tau_{aj} = \tau_e$ as in (20) is optimal (substitute (27) into (5), simplify by setting consumer emissions and abatement to zero, and modify (11) accordingly).

By the same argument, if two producers of j are active but have different technologies and emission factors, their emissions are taxed at the same rate in optimum. Note that two (or more) firms with different emission functions and different emission factors can be active at only one level of the emission tax, since if the emission tax is raised slightly, the producer with the higher emission factor shuts down (by the envelope theorem). However, if there are latent technologies, then at any emission tax level technologies with different emission factors can be active, and we have shown they shall be taxed at the same rate per unit of emissions. Thus, the marginal cost of abatement per unit of emissions reduced will be the same in productive sectors as amongst consumers:

$$(28) \quad \frac{-1}{f_{ja}} = \frac{-1}{f_{ia}} = \frac{-1}{f_{jb}} = \frac{\sum_h \beta^h \alpha^h}{\mu}.$$

For government, the Lagrangian (11) assumed that the government consumption vector x^P as well as the government abatement vector b^P was given. Modify (11) to reflect a choice of abatement, and partially differentiate with respect to b_j^P in addition to the previously applied instruments t_{e_j}, τ_{e_j} . No changes in expressions are implied for the previously established set of first order conditions. For the additional first order conditions reflecting optimal abatement for government, we have for all $j=1, \dots, n$:

$$(29) \quad \sum_h \beta^h \alpha^h \frac{de}{db_j^P} - \mu x_j^P = 0.$$

Using $\frac{de}{db_j^P} = f_{jb}^P x_j^P$, from (2) and (3) we can see that

$$(30) \quad \frac{\sum_h \beta^h \alpha^h}{\mu} = -\frac{1}{f_{jb}^P}.$$

Thus, in optimum, the marginal costs of abatement per unit of emissions reduced will be equalized across firms, government and households.

As a matter of implementation, if government agencies are geared to pursue their respective goals while maximizing some appropriate 'profit' function, then these agencies

should be exposed to the same emission tax (or abatement requirements) as the one levied on consumers and firms.

We have now shown that abatement should be stimulated by the same emission tax when abatement reduces own emissions. The generalization remaining is to allow abatement at any stage to influence emissions or abatement opportunities at other stages as well (as when the car's emissions can be reduced by abatement efforts in the car-factory, at the service station, in the oil refinery and by the driver). It is intuitive, now, that the emission tax (20) provides optimum stimulus in this more general setting. As producers and consumers join forces to minimize private costs – including emission taxes, emission reductions are provided effectively. Showing this involves additional notation, and is left to the reader²⁹.

We are now ready to summarize our findings:

Summary of central findings: Pigovian principles and production efficiency

Assume constant returns to scale, that the environmental good is separable from other goods, that within each activity consumers have uniform emission functions, that linear taxes on inputs, outputs and emissions can be differentiated by commodity (or that emission standards can be differentiated by commodity), and that different regimes can apply for consumers, producers and government.

i) Welfare optimum is characterized by the marginal rates of transformation between abatement and emission reductions a) equal across polluting activities (i.e. goods, sectors, $j \in N$), b) equal for emissions from consumers, producers and government, and c) equal to the welfare weighted sum of willingness to pay across consumers

²⁹ It may be worthwhile to revisit with a practical perspective the issue of the untaxed good. Our formulation states that consumer abatement is through application of the untaxed good. It is this feature which allows equal rates of transformation between abatement and emission reductions to be implemented by one emission tax faced by producers and consumers (since producers face pretax prices). Assume now that leisure has to be the untaxed good and that consumers may abate emissions with leisure (using time to drive more carefully, or to perform more laborious laundry with less polluting detergents) and by changing filters, and that producers may abate by installing filters and through many other actions. If leisure is the untaxed good, and filters can be taxed at zero rates when used in abatement, then the efficient solution can be implemented by confronting consumers and producers with the same emission tax. If abatement cannot be taxed at zero rates (when used in abatement by consumers), then the optimal allocation – still equalizing the marginal costs of emission reductions – is implemented by a separate emission tax for consumers.

adjusted by the shadow price of public revenue: $-\frac{1}{f_{jb}} = -\frac{1}{f_{ja}} = -\frac{1}{f_{jb}^P} = \frac{\sum_h \beta^h \alpha^h}{\mu}$.

ii) When abatement is untaxed and emissions are observable, such abatement can be implemented by a tax levied uniformly on all emissions (20):

$$\tau_e = \frac{\sum_h \beta^h \alpha^h}{\mu}.$$

iii) An emission tax satisfying this formula combined with commodity taxes satisfying the formula for optimal commodity taxes in the traditional problem without externalities (21) implements the optimal allocation.

iv) When emissions are not taxable, but emission standards or abatement standards can be used, the same allocation can be implemented by a combination of emission standards (as in i), above) and commodity taxes that include presumptive Pigovian taxes, as in Sandmo, 1975.

v) When abatement cannot be induced by the planner, the optimal allocation is implemented by commodity taxes which include presumptive Pigovian taxes, as in Sandmo, 1975.

With pollution just from producers and government, the equality of marginal rates of transformation between abatement and emission reductions is a predictable consequence of Diamond and Mirrlees' (1973) result. They showed that the set of producers and government should be treated as one, to all have equal marginal rates of transformation between goods. This clearly should apply even when an additional input valued by consumers, the environment, is included in the model. Amongst the findings of Cremer and Gahvari (1999) is that an emission tax should apply uniformly across industries. We add that this holds also for pollution from government, from firms with heterogeneous emission functions, and from consumers - only the latter one of which does not follow almost directly from Diamond and Mirrlees' treatment. Also, we show how standards can take the place of emission taxes under some plausible restrictions on monitoring.

The result least to be expected, that production efficiency shall include pollution abatement amongst consumers, must be understood in a context of assumptions about available instruments. Also, that result depends on the assumption that consumers face

the same abatement opportunities, implying that they have the same emission factors when exposed to emission taxes. We relax this assumption in the section to follow.

IV. Non-Uniform Emission Functions

In the previous section, we established that producers shall be taxed uniformly on emission irrespective of whether they have uniform emission functions. For consumers, the analysis till now has assumed uniform emission functions. We now investigate the consequences of heterogeneity across consumers in emission functions, reintroducing individual superscripts for emission functions, and thus abatement: $f_j^g(b_j^g)$.

We should note that the additivity in the relationship between emissions and the environmental good (equation 3) is retained. What we now allow implies only that consumers may differ in terms of emissions per unit consumed of the polluting good. An alternative formulation - also important in practice - could be that polluters differ in the relationship between emissions and the environmental good (so the damages could differ per unit emitted, rather than per unit consumed, which is our formulation). Results are very similar in nature, though with qualifications regarding instrument availability. The reason is that an emission tax is still 'first best' with respect to environmental protection when emission functions differ. If damages per unit emitted are different, in contrast, the emission tax is first best only if each polluter can be taxed at the same rate per unit of damages. With presumptive Pigovian taxes levied on each unit of the polluting good, the parallel is more direct, since that instrument loses its first best properties in both formulations.

Uniformity of emission functions for a given commodity is more plausible the more narrowly one can define each polluting commodity. Examining the model, this is a question of whether consumption with different emission functions can be differentiated in the commodity tax structure. If consumption *can* be differentiated in the commodity tax structure, so that within each "commodity" uniform emission functions result, then the results of the previous section apply.

To give a practical example, assume first that emissions are taxable, and that car travel is more polluting when using leaded gasoline than when using unleaded, but with emission functions that are uniform amongst users of leaded gasoline, and amongst users of unleaded gasoline. If the two fuels *can* be taxed separately in the commodity tax

structure, then the results of the previous section apply. Assume in contrast, that emission functions differ by car or user characteristics (old versus new/young, male versus female). To the extent that commodity taxes cannot be conditioned on these (perhaps they could, if odometer charges were used), one has set the scene for the topic of this section³⁰.

We assume polluters are exposed to non-individualized linear instruments: commodity taxes and emission taxes or uniform abatement requirements, $b_j^h = \bar{b}_j$. Under regulation, then, abatement is uniform by assumption, and emission factors may differ if emission functions are heterogeneous. Under an emission tax, consumers equalize marginal abatement costs $-1/f_{jb}^h(b_j^h(\tau_{ej})) = -1/f_{jb}^g(b_j^g(\tau_{ej})) = \tau_{ej}$ (re equation (6)), and abatement as well as emission factors may differ if emission functions are heterogeneous.

As an important background, in a setting with costless redistribution and revenue generation, an emission tax is a first-best instrument even when emission functions are heterogeneous. In contrast, if the planner cannot tax emissions, then presumptive taxation of goods would be an imperfect instrument under heterogeneous emission functions. This difference should be on our mind as we set out to analyze the cases with and without emission taxes separately.

Taxation of emissions

Corresponding to (16) and (17), our first order conditions for optimum are, for all $i=1,...,n$:

$$(31) - \sum_{h \in H} \beta^h \left[x_i^h + \alpha^h \sum_{g \in H} \sum_j f_j^g x_{ji}^g \right] + \mu \sum_{g \in H} \left[x_i^g + \sum_j (t_j + \tau_{ej} f_j^g) x_{ji}^g \right] = 0, \text{ and}$$

$$(32) - \sum_{h \in H} \beta^h \left[f_i^h x_i^h + \alpha^h \sum_{g \in H} \left(f_{ib}^g b_{ir}^g x_i^g + \sum_{j \in N} f_j^g x_{ji}^g f_i^g \right) \right]$$

³⁰ Empirical aspects of vehicle characteristics and emission factors are well known amongst practitioners, and were recently documented and discussed in Harrington, 1997. In Eskeland and Kong, 1998, the distributional consideration is examined in detail. One stylized fact found is that the expansion path in household energy use is toward energy carriers with lower emission factors (say, from coal and wood to electricity and natural gas). Another is that emission factors are lower for newer equipment, both because designs improve with vintage and because of deteriorating functions (emission control, combustion).

$$+ \mu \sum_{g \in H} \left[f_i^g x_i^g + \tau_{ei} (f_{ib}^g b_{ir}^h x_i^g) + \sum_j (t_j + \tau_{ej} f_j^g) x_{ji}^g f_i^g \right] = 0.$$

The reader may verify that straightforward application of Pigovian principles

($\tau_{ei} = \sum \beta^h \alpha^h / \mu$, all i) leaves (31) solved by commodity taxes satisfying the formula for optimal commodity taxes in the traditional problem without pollution, but that this emission tax is inconsistent with solving the set as a whole with only n remaining instruments. Thus, emission taxes cannot in general comply with Pigovian principles when emission functions are heterogeneous. We proceed to qualify and interpret these deviations from Pigovian principles.

Without loss of generality, let us split the taxes levied on emissions in (31) and (32) in two parts: one 'environmental tax' τ_e which we set according to Pigovian principles ($\tau_e = \sum \beta^h \alpha^h / \mu$), and a supplementary emission tax (or subsidy) τ_i which we leave for further investigation:

$$(33) \quad \tau_{ei} \equiv \tau_e + \tau_i,$$

$$(34) \quad \tau_e = \sum_h \beta^h \alpha^h / \mu.$$

Also, to simplify exposition, let us introduce the following expression (it is the derivative of revenue from consumer g with respect to t_i , except the part $\tau_e \sum_g \sum_j f_j^g x_{ji}^g$):

$$(35) \quad \frac{\partial R^g}{\partial t_i} \equiv \left[x_i^g + \sum_j (t_j + \tau_{ej} f_j^g) x_{ji}^g \right].$$

Substituting (33), (34) and (35) into (31) and (32), we have, for all $i=1, \dots, n$,

$$(36) \quad \sum_h \left(\beta^h x_i^h - \mu \frac{\partial R^h}{\partial t_i} \right) = 0, \text{ and}$$

$$(37) \quad \sum_{h \in H} \left(\beta^h f_i^h x_i^h - \mu \left[\frac{\partial R^h}{\partial t_i} f_i^h + \tau_i f_{ib}^h b_{ir}^h x_i^h \right] \right) = 0.$$

(37) can be rewritten using averages and covariances across consumers as follows:

$$(38) \quad \bar{f}_i \sum_{h \in H} \left(\beta^h x_i^h - \mu \frac{\partial R^h}{\partial t_i} \right) - H \left[\mu \tau_i \overline{f_{ib} b_{ir} x_i} - \text{Cov}(\beta x_i, f_i) + \mu \text{Cov} \left(\frac{\partial R}{\partial t_i}, f_i \right) \right] = 0.$$

We can see that if the covariances in (38) are zero, then we can set $\tau_i = 0$, all i , and each equation in (38) is simply $\overline{f_i}$ times (36). Thus, when those covariances are zero, emissions are taxed according to Pigovian principles (34) only, and a commodity tax structure which solves (36) is optimal. When τ_i is zero for all polluting goods, (36) is also the solution to the traditional optimal commodity tax problem (i.e. without pollution).

More generally, let us observe that (38) is a sum of two terms, where the first is simply f times (36), so we may think of the optimal tax structure as follows. (36) is at most of rank n , so the n commodity tax rates can be reserved to solve (36), conditional on a set of supplementary emission tax rates. Thus, the system (36) and (38) has a solution for which the supplementary emission tax rates render zero the bracket term in (38). Assuming that $\mu \overline{f_{ib} b_{ir} x_i} \neq 0$, and using

$Cov(f_i, \partial R / \partial t_i) = Cov(f_i, x_i) + \sum_j t_j Cov(f_i, x_{ji}) + \sum_j \tau_j Cov(f_i, f_j x_{ji})$, the term in brackets of (38) is zero for

$$(39) \quad \tau_i = \frac{Cov(f_i, \beta x_i) - \mu \left[Cov(f_i, x_i) + \sum_j t_j Cov(f_i, x_{ji}) + \sum_j \tau_j Cov(f_i, f_j x_{ji}) \right]}{\mu \overline{f_{ib} b_{ir} x_i}},$$

all $i=1, \dots, n$). This is no explicit solution: not only are there tax rates on the right hand side, but all the expressions may be functions of the tax rates. Nevertheless, from (39) we learn that it is a specific set of covariances that gives a potential role to “non-Pigovian” supplementary taxation of emissions. To gain some additional insight, let us make the assumption that only one activity is polluting: $\overline{f_j} = 0$, $j \neq i$, and assume that

$$1 + \frac{\mu Cov(f_i, f_i x_{ii})}{\overline{f_{ib} b_{ir} x_i}} \neq 0:$$

$$(40) \quad \tau_i = \frac{Cov(f_i, \beta x_i) - \mu \left[Cov(f_i, x_i) + \sum_j t_j Cov(f_i, x_{ji}) \right]}{\mu (\overline{f_{ib} b_{ir} x_i} + Cov(f_i, f_i x_{ii}))}.$$

(40) is still a complicated combination of effects, but all intuitively play a role given that the social planner compares the effects of supplementary emission taxes to the effects of commodity taxes. The two terms in the denominator represent the responsiveness of

abatement and emissions to the emission tax. These responses are, from a first-best perspective, wasteful when emission taxes differ from Pigovian principles, so the absolute value of their sum *ceteris paribus* reduces the value of supplementary emission taxes. In the numerator, the first covariance represents the planner's evaluation of the distributive pattern of the emission tax (as compared to the commodity tax). As an example, assume that the denominator is negative ($\overline{f_{ib} b_{ir} x_i} < 0$ by assumption) and that the bracket term is zero. If the emission factor falls with income (as when wealthier have newer cars and these are less polluting) and βx_i falls (rises) with income, the supplementary non-Pigovian emission tax will be negative (positive).³¹

The combined term in brackets distinguishes between the emission tax and the commodity tax in terms of the marginal effect on revenue. If the bracket term is negative, then it means that increasing the emission tax on good i raises revenue less than f_i times a change in t_i , an effect which *ceteris paribus* reduces the emission tax (assuming the denominator < 0).

To focus on revenue and redistributive considerations, assume that the denominator is negative and the tax weighted term with demand responsiveness in (40), $\sum_j t_j \text{Cov}(f_i, x_{ji})$ is zero: For τ_i to have a determined sign *a priori*, $\text{Cov}(f_i, \beta x_i)$ and $\text{Cov}(f_i, x_i)$ must be of opposite sign. For a normal good, a “steep” β is sufficient to ensure a sign, and the sign is given by whether f_i is increasing or declining with income. Giving a practical illustration, emission factors will often be declining in income. Assuming β steep enough that $\text{Cov}(f_i, \beta x_i)$ is positive even though $\text{Cov}(f_i, x_i)$ is negative, these effects lead to a downward adjustment in emission taxes from Pigovian levels.

Let us finally focus on the possible covariance between the emission factor and demand responsiveness. To illustrate simply, assume that in (40) all cross price elasticities are zero, and that $\text{Cov}(x_{ii}, f_i) > 0$, so that the more polluting consumers are less responsive in their demand. Assume further that the denominator is negative and that

³¹Intuition: In case βx_i and f_i fall with income, the poor are hurt more by the emission tax than by the commodity tax. A slight change in taxation from emissions to the commodity redistributes from rich to poor.

the other covariances are zero. If t_i is positive (negative), then non-Pigovian taxation of emissions is positive (negative). In this case, the planner takes the opportunity for ‘price discrimination’ simply to reduce distortions (this effect does not depend on the vector β): More polluting consumers are less price responsive (for good i) and should therefore face a higher effective price for reasons well known in the literature (Ramsey-pricing). A distortionary effect of this is that emissions are taxed ‘too heavily’, so ‘too much abatement’ is executed.

We should highlight again that these perspectives often will point us back to ask for a more differentiated commodity tax structure, rather than to actually modifying emission taxes with non-Pigovian objectives. When emission taxes are brought to differ from Pigovian principles, here, it is because they take on roles in redistribution and revenue generation that are left unsolved by other instruments. Using emission taxes for these purposes have separate, identifiable costs, and can be attractive only if other available instruments entail costs as well.

Presumptive Pigovian taxes

In the case of non-uniform emission functions, taxation of commodities in presumption of emissions has a weaknesses in addition to the potential weakness of not inducing abatement: consumption by dirtier consumers and cleaner consumers is discouraged with ‘equal pressure’. This may present a problem of fairness and distribution, but also of efficiency, since the emission factor determines the emission reductions ‘bought’ when consumption is reduced.

Let us initiate this analysis by assuming that emission factors are given. This problem is similar to the problem of imperfect corrective pricing analyzed by Diamond, 1973³². Our results also compare with several studies on taxation of substitutes and

³² In the literature, ‘corrective taxation’ is used synonymously with ‘Pigovian taxation’, and corrective pricing refers to prices that include corrective elements, equivalent to prices that include our ‘presumptive Pigovian taxes’. Diamond’s problem is more general than ours in the sense that he makes no separability assumption, so his ‘public good’ (absence of congestion) may influence demand. On the other hand, our problem is more general in including distortionary revenue generation, and in including effects across markets in the external effects as well. These differences are illustrated in one of his concluding passages: “..the optimal surcharge will be small relative to the average externality when individuals who contribute greatly to congestion per unit demanded .. tend to have demands which are congestion sensitive .. and price insensitive ..”. In our model, the analogue to congestion sensitivity is zero due to separability (the case for

complements to externality-creating goods when the ideal corrective instrument is not available³³. Analogously to (16), first order conditions for optimal commodity taxes are:

$$(41) \quad -\sum_h \beta^h \left[x_i^h + \alpha^h \sum_g \sum_j f_j^g x_{ji}^g \right] + \mu \left[\sum_g x_i^g + \sum_j (t_j + t_{ej}) x_{ji}^g \right] = 0, i=1, \dots, n.$$

In (41), we have followed steps in previous sections to ‘artificially’ split the commodity tax rates in two parts. The system is then indeterminate, and we can choose one part arbitrarily. Let us choose t_j , $j = 1, \dots, n$ such as to solve the traditional problem of optimal commodity taxes when there is no pollution:

$$(42) \quad -\sum_h \beta^h x_i^h + \mu \sum_g \left[x_i^g + \sum_j t_j x_{ji}^g \right] = 0, \text{ all } i = 1, \dots, n.$$

Then, for (41) to be solved, we must have

$$(43) \quad \sum_h \beta^h \alpha^h \sum_g \sum_j f_j^g x_{ji}^g = \mu \sum_j t_{ej} \sum_g x_{ji}^g, i = 1, \dots, n.$$

Assuming that the coefficient matrix in (43) is nonsingular, we may use Cramer’s rule to develop more explicit expressions. For the two-good case the presumptive Pigovian tax on good one is

$$(44) \quad t_{e1} = \left(\frac{\sum_g f_1^g (x_{11}^g x_{22} - x_{12}^g x_{21})}{H(x_{11} x_{22} - x_{12} x_{21})} + \frac{\sum_g f_2^g (x_{21}^g x_{22} - x_{22}^g x_{21})}{H(x_{11} x_{22} - x_{12} x_{21})} \right) \frac{\sum_h \beta^h \alpha^h}{\mu},$$

where the denominator is positive.

Corresponding formulas, for t_{e2} or for systems with more goods are straightforward to derive. We may consider a system consisting of traditional commodity taxes (42) and presumptive Pigovian taxes (44) as a generalized version of commodity taxes which include presumptive Pigovian taxes (19). The large parenthesis in (44) then plays the role of the emission factor f_1 , and the first fraction in this parenthesis is indeed a weighted average for f_1 , which equals \bar{f}_1 if the covariances $Cov(f_1, x_{11})$ and $Cov(f_1, x_{12})$ are zero (as when f_1 is uniform). The second fraction in the parenthesis represents emission spillovers via cross-price elasticities to good 2, and is zero if the

such sensitivity is more compelling for congestion), and our results with respect to price sensitivity will be less clear cut, due to cross price effects both in revenue generation and in external effects.

³³ Noteable contributions are: Green and Sheshinski, 1976; Sandmo, 1976; Balcer, 1980; Wijkander, 1985, Greenwald and Stiglitz, 1986.

covariances $Cov(f_2, x_{21})$ and $Cov(f_2, x_{22})$ are zero. Interestingly, good 1 may be taxed with reference to the Pigovian objective even if only good 2 is polluting, a result observed in the literature on indirect instruments (see below). To focus on covariances, let us rearrange. (44) \Rightarrow

(45)

$$t_{e1} = \left(\overline{f_1} + \frac{\overline{x_{22}}[Cov(f_1, x_{11}) + Cov(f_2, x_{21})] - \overline{x_{21}}[Cov(f_1, x_{12}) + Cov(f_2, x_{22})]}{\overline{x_{11}x_{22}} - \overline{x_{12}x_{21}}} \right) \frac{\sum \beta^h \alpha^h}{\mu}.$$

The four covariances all are between an emission factor and the price responsiveness of the good to which it applies. If we assume that own price elasticities are negative and cross price elasticities are positive, then positive covariances result in a tax rate lower than under unweighted average emission factors. The reason for this is that positive covariances raise the marginal costs of emission reductions relative to that indicated by the average coefficient (illustration: $Cov(f_1, x_{11}) > 0 \Rightarrow Cov(f_1, |x_{11}|) < 0$, so individuals with high emission factors adjust consumption less than average).

To enhance intuition further and to compare with the literature on indirect Pigovian instruments (see above), let us consider again the case with two taxed goods and assume that good 1 is not polluting. The Pigovian parts of the commodity tax rates in this context are

$$(46) \quad t_{e1} = \frac{\overline{x_{22}}Cov(f_2, x_{21}) - \overline{x_{21}}Cov(f_2, x_{22})}{\overline{x_{11}x_{22}} - \overline{x_{12}x_{21}}} \frac{\sum \beta^h \alpha^h}{\mu}, \text{ for the nonpolluting good, and}$$

$$(47) \quad t_{e2} = \left[\overline{f_2} + \frac{\overline{x_{11}}Cov(f_2, x_{22}) - \overline{x_{12}}Cov(f_2, x_{21})}{\overline{x_{11}x_{22}} - \overline{x_{12}x_{21}}} \right] \frac{\sum \beta^h \alpha^h}{\mu}, \text{ for the polluting good.}$$

Our model has greatest similarity with that of Balcer, 1980. The literature we refer to does not include distortionary revenue generation, but compares with our formula for the presumptive Pigovian tax which bares evidence of distortionary taxation only through $\sum \beta \alpha / \mu$. Balcer focuses on the dimensions of “large offenders” (consumers g , for whom $f_2^g > \overline{f_2}$, in our terminology) and “large offender complementarity” ($Cov(f_2, x_{21}) < 0$).

If we make the assumption of 'aggregate independence' in demand ($\overline{x_{12}} = \overline{x_{21}} = 0$), we can tabulate results analogous to some of those in Balcer's table 1:³⁴

Table 1: Presumptive Pigovian Taxation under Aggregate Independence ($\overline{x_{12}} = \overline{x_{21}} = 0$)			
Assumption for "large offenders":	Own -Price Responsive $Cov(f_2, x_{22}) < 0$	Own-price Neutral $Cov(f_2, x_{22}) = 0$	Own-Price Non-responsive $Cov(f_2, x_{22}) > 0$
Result for Direct Instrument	$t_{e_2} > \frac{\overline{f_2} \sum \beta \alpha}{\mu}$	$t_{e_2} = \frac{\overline{f_2} \sum \beta \alpha}{\mu}$	$t_{e_2} < \frac{\overline{f_2} \sum \beta \alpha}{\mu}$
Assumption for "large offenders"	Complementarity $Cov(f_2, x_{21}) < 0$	Neutrality $Cov(f_2, x_{21}) = 0$	Substitutability $Cov(f_2, x_{21}) > 0$
Result for Indirect Instrument	$t_{e_1} > 0$	$t_{e_2} = 0$	$t_{e_2} < 0$

The results under aggregate independence are quite intuitive. As examples, for the direct instrument t_{e_2} the tax is raised by own - price responsiveness for large offenders, since this reduces the costs of emission reductions when the price is raised equally for all. For the indirect instrument, t_{e_1} , the level will be negative if there is large offender substitutability, since a subsidy then reduces consumption of good 2 amongst large offenders but not for average offenders, thus providing emission reductions at low costs. As an illustration, assume one group (say, the young) would substitute metro- for car travel if metro fares were lower, but that for the old the two are complements, so that aggregate demand for car travel is independent of metro-fares. If young people pollute more than old when traveling by car, the commodity tax rate for metro travel would be adjusted downwards for Pigovian reasons³⁵.

³⁴ Note: Additional results in the absence of aggregate independence are found by examining (46) and (47). Examples: Average complementarity ($\overline{x_{12}} < 0$, $\overline{x_{21}} < 0$): the direct instrument t_{e_2} will be raised (reduced), if there is large offender substitutability (complementarity). The indirect instrument t_{e_1} will be raised (reduced) if large offenders are own-price nonresponsive.

³⁵ Little systematic knowledge exists about demand responsiveness for polluting goods, let alone for disaggregate groups. In practical discussions of the air pollution control program for Mexico City (Eskeland, 1994), the responsiveness of demand for travel, including how it might vary by groups of vehicles, was one of the issues on the agenda. Eskeland and Feyzioglu (1997b) estimated responsiveness in demand for gasoline and vehicles in Mexico. Eskeland and Feyzioglu (1997) found very unfortunate consequences of a scheme to ration trips in Mexico City: Households bought additional, used cars at

Our results for the indirect instrument, t_{e_1} , are similar to those of Balcer, though his results are sharper due to more restrictive assumptions³⁶. For the direct instrument, t_{e_2} , we report how the tax level compares to $\overline{f_2} \sum \beta^h \alpha^h / \mu$, whereas Balcer compares to the tax level without any taxation of the associated good (i.e. $t_{e_1}=0$). For this reason, his results are not qualified by own-price responsiveness (x_{22} and $Cov(f_2, x_{22})$).

Emission standards and commodity taxes including presumptive Pigovian taxes

Equation (41) defines optimal commodity taxes including presumptive Pigovian taxes for any given abatement levels. If we assume that the planner can regulate abatement but must do this uniformly for all consumers (though emission functions differ), then first order condition for optimal abatement are, for all $i=1, \dots, n$:

$$(48) \quad - \sum_n \beta^h \left[x_i^h + \alpha^h \sum_g \left(f_{ib}^g x_i^g + \sum_j f_j^g x_{ji}^g \right) \right] + \mu \sum_g \left[\sum_j (t_j + t_{ej}) x_{ji}^g \right] = 0.$$

Commodity taxes are optimal, so we may subtract (43), to obtain

$$(49) \quad - \sum_n \beta^h \alpha^h \sum_g f_{ib}^g x_i^g - \mu \sum_g x_i^g = 0.$$

This results in ($f_{ib}^g x_i^g < 0$ by assumption)

$$(50) \quad \frac{\sum_h \beta^h \alpha^h}{\mu} = - \frac{\sum_g x_i^g}{\sum_g f_{ib}^g x_i^g} = \frac{-\overline{x_i}}{x_i f_{ib} + Cov(f_{ib}, x_i)},$$

which simplifies to (26) $\sum_n \beta^h \alpha^h / \mu = -1/\overline{f_{ib}}$ if the covariance between consumption of the polluting good and the marginal cost of abatement is zero. (50) reflects a rather intuitive relationship between the instrument at hand and the costs of emission reductions. The planner has to ask high consumption individuals to abate proportionally more than average consumption individuals (they must abate equally per unit). If the covariance $Cov(f_{ib}, x_i)$ is negative (positive), so that high – consumption individuals have low (high) marginal costs, then optimum is found in a point with – *ceteris paribus* –

unexpected rates to circumvent the regulation. Initially, the program rationing car use was seen as politically attractive because of its distributional implications.

³⁶ Balcer has zero income effects, so $x_{12} = x_{21}$. His results on the indirect instrument are not qualified by aggregate independence.

higher (lower) abatement standards and lower (higher) emissions than if the covariance were zero.

Concluding, when presumptive taxes and standards are used, covariances with demand patterns influence instruments, but only for reasons related to efficiency in environmental protection. The demand patterns are important for efficiency because - for both instruments - they determine the marginal costs of reducing emissions when, these instruments are unable to equalize costs across consumers. The reason why distributional considerations do not directly affect these instruments under nonuniform emission functions (though they do for emission taxes) is simply that in this model neither instrument can do anything towards redistribution that commodity taxes cannot.

V. Discussion

Our aim with this study was to examine whether intuition about ‘pricing’ the environment applies in more general contexts than explored earlier. Does Sandmo’s ‘additivity property’ (1975) apply in such a way that different polluting activities be treated in the same fashion? If negative externalities can be reduced not only by changes in consumption patterns, but also by making each activity cleaner (abatement efforts), how shall optimal policy combine inducements to these various approaches? Finally, if negative externalities are caused by agents as different as consumers, producers and government, how does optimal policy combine efforts from these to reduce pollution?

Three assumptions are critical when we show that the marginal costs of emission reductions, per unit of emissions, shall be the same across activities (goods, sectors), and across polluters. The assumption of constant returns to scale is widely applied in the literature, and is required in the present context since we want to see how established results on production efficiency extend. Second, we assume that consumers have equal access to pollution abatement opportunities (but also examine results of relaxing this assumption). Third, we assume that the planner can differentiate his policy instruments (emission taxes or abatement standards) by polluting good, and by whether the polluter is a consumer, a producer or government, but he cannot differentiate such instruments - or the commodity taxes - by personal characteristics, or make them non-linear in individual emissions.

Comparing our results with Sandmo's results, they represent generalizations that are very simple at a formal level: One may replace the presumptive Pigovian part of his commodity tax rates with an emission tax applied uniformly across agents and goods: The emission factors that are part of the expression for Sandmo's tax rates will now form the base of an emission tax. Such a tax, combined with commodity taxes that satisfy the formula for optimal taxation in the traditional problem without external effects, induces optimum substitution towards less polluting activities as well as optimal abatement everywhere.

The paper adds that the applicability of these principles is not limited to contexts in which emissions are monitored at the source. Emission standards (or abatement standards) may be implemented with more limited monitoring capabilities (car model certifications, for instance), and a combination of emission standards and commodity taxes that include presumptive Pigovian taxes can under the applied assumptions implement the same allocation as the one implementable by commodity taxes and emission taxes.

The results also extend the production efficiency result of Diamond and Mirrlees, to include efficiency in environmental protection. For polluters that are producers and in government, production efficiency (in a sense including equal marginal costs of emission reductions) is to be expected. As an additional public good – the environment – is included in the relevant input-output vector, the result that the optimal vector is at the aggregate production frontier prevails.

When production efficiency applies also for polluting consumers – in the sense that they too shall abate pollution at the same marginal rate of transformation as firms and government – it is more surprising. One might expect that the planner – in his desire to redistribute or collect revenue at minimal distortionary costs -would choose to apply different pressures to abate pollution in different activities in order to pursue these goals. When consumers have equal access to abatement technology, however, emission taxes differentiated according to polluting activity (i.e. goods) are redundant instruments for redistribution and revenue generation: they have the same dimensionality as commodity taxes, and commodity taxes dominate since they do not induce additional resource costs by making abatement deviate from efficient patterns.

When emission functions differ across producers, no deviation from Pigovian principles result. The consequences when emissions functions are heterogeneous across consumers are of two kinds, both related to covariances between emission factors and consumption patterns and demand responses. First, when emission taxes are available, the planner has a Pigovian instrument that is first best, and deviations from Pigovian principles come if the differentiated pattern of emission taxes and abatement costs lend themselves to his goals of redistribution and revenue generation. Second, when emission taxes are not available but emission standards are used, non-uniformity of emission functions influence policy because the instruments are no longer first best from a Pigovian perspective. These resulting adjustments are related merely to the objective of correcting externalities at least costs – not to revenue generation or redistribution. As an example, if for a good j marginal costs of emission reductions covary negatively with consumed quantities, then this enhances the cost effectiveness of the emission standard, relative to the case with no covariance.

Finally, simplicity in principles in this case also seems to simplify implementation. Think about how to stimulate pollution reductions from those making cars, roads, tires and fuels, and those using cars. First, it simplifies implementation that the stimulus given to abatement at one stage (say at the factory) is independent of whether the abatement yields i) emission reductions at that stage (the factory), ii) emission reductions at some other stage (in the refinery, in the commute), or iii) enhanced abatement opportunities at some other stage (the refinery, the commute). This allows abatement efforts at all stages optimally to be stimulated by a uniform emission tax levied where emissions occur. Second, it simplifies things that optimal abatement is independent of whether the car is used by government, firms or households, for weddings or for work.

Finally, principles could be helpful also in simplifying the organization of intervention for revenue and environmental protection, and perhaps in reducing the scope for wasteful political battles in environmental policy making. As an illustration, notice that the emission tax that induces optimal abatement in its formula refers only to benefits of environmental protection, not to price elasticities for polluting goods. Nevertheless, it also induces optimal substitution towards less polluting goods, in the sense that this emission tax should be combined with commodity tax rates satisfying the formula for optimal taxation in the traditional problem without external effects. Thus, at a very

intuitive level, the environmental minister is concerned about pricing the environment - and the finance minister may think about him as such. The revenues will contribute to the general treasury and thereby influence the shadow price of public revenue. *Thereby*, the environmental minister's agenda influences the optimal commodity tax problem of the finance minister, but the finance minister need not think about the environmental costs or opportunities in each activity. Similarly, the environmental minister need not think about whether he - when taxing polluting sectors - tax sectors that are important for other reasons, such as revenue or redistribution.

REFERENCES

- Atkinson, Anthony B. and Joseph E. Stiglitz. 1976. "The Design of Tax Structure: Direct Versus Indirect Taxation", *Journal of Public Economics*, Vol. 6, 55-75.
- _____. 1980. "Lectures on Public Economics", McGraw-Hill Book Company, New York.
- Atkinson, Anthony B. and N.H. Stern. 1974. "Pigou, Taxation and Public Goods", *Review of Economic Studies*, Vol. 41, No. 1, 119-128.
- Auerbach, Alan, J. 1985. "The Theory of Excess Burden and Optimal Taxation", In *Handbook of Public Economics*, edited by Auerbach and Martin Feldstein, Vol. 1, Chap. 2, North-Holland, New York.
- Balcer, Yves. 1980. "Taxation of Externalities: Direct Versus Indirect", *Journal of Public Economics*, Vol. 13, 121-129.
- Baumol, William J., and Wallace E. Oates. 1988. "The Theory of Environmental Policy", Cambridge University Press, Cambridge, UK, 2nd Edition.
- Bergstrom, Theodore C. 1989. "A Fresh look at the Rotten Kid Theorem- and Other Household Mysteries." *Journal of Political Economy*, Vol. 97, No. 5, 1138-59.
- Bovenberg, A. Lans and Lawrence H. Goulder. 1996. "Optimal Environmental Taxation in the Presence of Other Taxes: General-Equilibrium Analyses", *American Economic Review*, Vol. 86, No. 4, 985-1000.
- Bovenberg, A. Lans and F. van der Ploeg. 1994. "Environmental Policy, Public Finance and the Labour Market in a Second-Best World", *Journal of Public Economics*, Vol. 55, No. 3, 349-390.
- Buchanan, James M. and Gordon Tullock. 1975. "Polluters' Profits and Political Response: Direct Controls versus Taxes", *American Economic Review*, Vol. 65, No. 1, 139-147.
- Corlett, W. J. and D. C. Hague. 1953. "Complementarity and the Excess Burden of Taxation", *The Review of Economic Studies*, Vol. 21, 21-30.
- Crandall, R.W., H. Gruenspecht, T. Keller and L. Lave. 1986. "Regulating the Automobile", The Brookings Institution, Washington, D.C.
- Cremer, Helmuth, Firouz Gahvari and Norbert Ladoux. 1998. "Externalities and Optimal Taxation", *Journal of Public Economics*, Vol. 70, 343-364.

- Cremer, Helmuth and Firouz Gahvari. 1999. "What to Tax: Emissions or Polluting Goods?", Mimeographed, University of Toulouse, Toulouse, France .
- Cropper, Maureen L. and Wallace E. Oates. 1992. "Environmental Economics: A Survey", *Journal of Economic Literature*, Vol. 30, No. 2, 675-740.
- Diamond, Peter A. and James A. Mirrlees. 1971. "Optimal Taxation and Public Production I: Production Efficiency", *American Economic Review*, Vol. 61, 8-27.
- Diamond, Peter A. 1973. "Consumption Externalities and Imperfect Corrective Pricing", *The Bell Journal of Economics and Management Science*, Vol. 4, No. 2, 526-538.
- _____. 1975. "A Many-Person Ramsey Tax Rule", *Journal of Public Economics*, Vol. 4, 335-342.
- Eskeland, Gunnar S. and Emmanuel Jimenez. 1992. "Policy Instruments for Pollution Control in Developing Countries", *World Bank Research Observer*, Vol. 7, No. 2, 145-169.
- Eskeland, Gunnar S. 1994. "A Presumptive Pigovian Tax: Complementing Regulation to Mimic an Emissions Fee", *The World Bank Economic Review*, Vol. 8, No. 3, 373-394.
- Eskeland, Gunnar S. and Shanta Devarajan. 1995. "Taxing Bads by Taxing Goods: Pollution Control with Presumptive Charges", The World Bank, Washington, D.C.
- Eskeland, Gunnar S. and Tarhan Feyzioglu. 1997. "Rationing Can Backfire: The 'Day without a Car' in Mexico City", *World Bank Economic Review*, Vol. 11, No. 3, 383-408.
- Eskeland, Gunnar S. and Tarhan Feyzioglu. 1997b. "Is Demand for Polluting Goods Manageable? An Econometric Study of Car Ownership and Use in Mexico", *Journal of Development Economics*, Vol. 53, No.2, 423-445.
- Eskeland, Gunnar S. and Chingying Kong. 1998. "Protecting the Environment and the Poor: A Public Goods Framework Applied to Indonesia", Policy Research Working Paper Series, No. 1961. The World Bank, Washington, D.C.
- Gillis, Malcolm. 1989. "Tax Reform in Developing Countries" (editor), Duke University Press, Durham, N.C.
- Goulder, Lawrence H., I. Parry, R. Williams III and D. Burtraw. 1999. "The Cost-Effectiveness of Alternative Instruments for Environmental Protection in a Second-Best Setting", *Journal of Public Economics*, Vol. 72, No. 3, 329-360.

- Green, Jerry and Eytan Sheshinski. 1976. "Direct Versus Indirect Remedies for Externalities", *Journal of Political Economy*, Vol. 84, No. 4, 797-808.
- Greenwald, Bruce C. and Joseph E. Stiglitz. 1986. "Externalities in Economies with Imperfect Information and Incomplete Markets", *Quarterly Journal of Economics*, Vol. 101, No. 2, 229-264.
- Harrington, Winston . 1997. "Fuel Economy and Motor Vehicle Emissions", *Journal of Environmental Economics and Management*, Vol. 33, 240-252.
- Meade, J. E. 1952. "External Economies and Diseconomies in a Competitive Situation", *The Economic Journal*, Vol. 62, No. 245, 54-67.
- Mirrlees, J. A. 1971. "An Exploration in the Theory of Optimum Income Taxation", *Review of Economic Studies*, Vol. 38, No. 2, 175-208.
- Munk, Knud Joergen. 1978. "Optimal Taxation and Pure Profit", *Scandinavian Journal of Economics*, Vol. 80, No. 1, 1-19.
- Musgrave, Richard A. and Peggy B. Musgrave. 1984. "Public Finance in Theory and Practice", McGraw-Hill, New York, 4th edition.
- Nelson, R. A., T. Tietenberg and M. R. Donihue. 1993. "Differential Environmental Regulation: Effects on Electric Utility Capital Turnover and Emissions", *The Review of Economics and Statistics*, Vol. 75, No. 2, 368-373.
- Newbery, David and Nicholas Stern. 1987. "The Theory of Taxation for Developing Countries" (editors), Oxford University Press, New York.
- Pigou, Arthur Cecil. 1947. "A Study in Public Finance", MacMillan & Co. LTD, London. 3rd edition, 1949 reprint.
- Ramsey, F. P. 1927. "A Contribution to the Theory of Taxation", *The Economic Journal*, Vol. 37, No. 147, 47-61.
- Samuelson, Paul A. 1951. "Theory of Optimal Taxation", Memorandum to the U.S. Treasury. Reprinted in *Journal of Public Economics*, 1986, Vol. 30, 137-143.
- _____ 1954. "The Pure Theory of Public Expenditure", *Review of Economics and Statistics*, Vol. 36, 387-389.
- Sandmo, Agnar. 1975. "Optimal Taxation in the Presence of Externalities", *Swedish Journal of Economics*, Vol. 77, 86-98 (the journal is now published under the name *Scandinavian Journal of Economics*).
- _____ 1976. "Optimal Taxation – An Introduction to the Literature", *Journal of Public Economics*, Vol. 6, 37-54.

1976b. "Direct Versus Indirect Pigovian Taxation", *European Economic Review*, Vol. 7, 337-349.

Schmutzler, Armin and Lawrence H. Goulder. 1997. "The Choice between Emission Taxes and Output Taxes under Imperfect Monitoring", *Journal of Environmental Economics and Management*, Vol. 32, 51-64.

Tietenberg, Tom. 1992. "Environmental and Natural Resource Economics", Harper Collins Publishers, New York, 3rd edition.

Weitzman, M. L. 1974. "Prices vs. Quantities", *Review of Economic Studies*, Vol. 41, 477-491.

Wijkander, Hans. 1985. "Correcting Externalities through Taxes on Subsidies to Related Goods", *Journal of Public Economics*, Vol. 28, 111-125.

Wittman, Donald. 1989. "Why Democracies Produce Efficient Results.", *Journal of Political Economy*, Vol. 97, No. 6, 1395-1424.

Policy Research Working Paper Series

Title	Author	Date	Contact for paper
WPS2302 Why Liberalization Alone Has Not Improved Agricultural Productivity in Zambia: The Role of Asset Ownership and Working Capital Constraints	Klaus Deininger Pedro Olinto	March 2000	M. Fernandez 33766
WPS2303 Malaria and Growth	F. Desmond McCarthy Holger Wolf Yi Wu	March 2000	H. Sladovich 37698
WPS2304 Disinflation and the Supply Side	Pierre-Richard Agénor Lodovico Pizzati	March 2000	T. Loftus 36317
WPS2305 The Impact of Banking Crises on Money Demand and Price Stability	Maria Soledad Martinez Peria	March 2000	A. Yaptenco 31823
WPS2306 International Contagion: Implications for Policy	Roberto Chang Giovanni Majnoni	March 2000	E. Mekhova 85984
WPS2307 Surveying Surveys and Questioning Questions: Learning from World Bank Experience	Francesca Racanati Scott J. Wallsten Lixin Colin Xu	March 2000	P. Sintim-Aboagye 37644
WPS2308 How Small Should an Economy's Fiscal Deficit Be? A Monetary Programming Approach	Paul Beckerman	March 2000	H. Vargas 38546
WPS2309 What Drives Private Saving around the World?	Norman Loayza Klaus Schmidt-Hebbel Luis Servén	March 2000	E. Khine 37471
WPS2310 How Politics and Institutions Affect Pension Reform in Three Postcommunist Countries	Mitchell A. Orenstein	March 2000	M. Leenaerts 84264
WPS2311 The Buenos Aires Water Concession	Lorena Alcázar Manuel A. Abdala Mary M. Shirley	April 2000	P. Sintim-Aboagye 38526
WPS2312 Measuring Governance, Corruption, and State Capture: How Firms and Bureaucrats Shape the Business Environment in Transition Economies	Joel S. Hellman Geraint Jones Daniel Kaufman Mark Schankerman	April 2000	D. Bouvet 35818
WPS2313 How Interest Rates Changed under Financial Liberalization: A Cross-Country Review	Patrick Honohan	April 2000	A. Yaptenco 31823

Policy Research Working Paper Series

	Title	Author	Date	Contact for paper
WPS2314	Technological Leadership and Foreign Investors' Choice of Entry Mode	Beata K. Smarzynska	April 2000	L. Tabada 36896
WPS2315	Investment in Natural Gas Pipelines in the Southern Cone of Latin America	Alejandro Jadresic	April 2000	M. Salehi 37157
WPS2316	Distributional Outcomes of a Decentralized Welfare Program	Emanuela Galasso Martin Ravallion	April 2000	P. Sader 33902
WPS2317	Trade Negotiations in the Presence of Network Externalities	Keiko Kubota	April 2000	L. Tabada 36896
WPS2318	Regulatory Reform, Competition, and Innovation: A Case Study of the Mexican Road Freight Industry	Mark A. Dutz Aydin Hayri Pablo Ibarra	April 2000	H. Sladovich 37698